Subject Code - 041

Sample Question Paper
CLASS: XII
Session: 2021-22
Mathematics (Code-041)
Term - 1

Time Allowed: 90 minutes **Maximum Marks: 40**

General Instructions:

1. **This question paper contains three sections – A, B and C. Each part is compulsory.**
2. **Section - A has 20 MCQs, attempt any 16 out of 20.**
3. **Section - B has 20 MCQs, attempt any 16 out of 20**
4. **Section - C has 10 MCQs, attempt any 8 out of 10.**
5. **All questions carry equal marks.**
6. **There is no negative marking.**

SECTION – A

In this section, attempt any 16 questions out of Questions 1 – 20.
Each Question is of 1 mark weightage.

1. sin [$\frac{\pi}{3}$ −$\sin^{-1}$ ($-\frac{1}{2}$)] is equal to: — 1

a) $\frac{1}{2}$	b) $\frac{1}{3}$
c) -1	d) 1

2. The value of k (k < 0) for which the function f defined as — 1

$$f(x) = \begin{cases} \frac{1-\cos kx}{x\sin x}, & x \neq 0 \\ \frac{1}{2}, & x = 0 \end{cases}$$

is continuous at $x = 0$ is:

a) ± 1	b) -1
c) $\pm\frac{1}{2}$	d) $\frac{1}{2}$

3. If A = [a_{ij}] is a square matrix of order 2 such that $a_{ij} = \begin{cases} 1, & when\ i \neq j \\ 0, & when\ i = j \end{cases}$, then A^2 is: — 1

a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$	b) $\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}$
c) $\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$	d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

4. Value of k, for which A = $\begin{bmatrix} k & 8 \\ 4 & 2k \end{bmatrix}$ is a singular matrix is: — 1

a) 4	b) -4
c) ±4	d) 0

5. Find the intervals in which the function f given by f (x) = $x^2 - 4x + 6$ is strictly increasing: 1

a) $(-\infty, 2) \cup (2, \infty)$	b) $(2, \infty)$
c) $(-\infty, 2)$	d) $(-\infty, 2] \cup (2, \infty)$

6. Given that A is a square matrix of order 3 and | A | = - 4, then | adj A | is equal to: 1

a) -4	b) 4
c) -16	d) 16

7. A relation R in set A = {1,2,3} is defined as R = {(1, 1), (1, 2), (2, 2), (3, 3)}. Which of the following ordered pair in R shall be removed to make it an equivalence relation in A? 1

a) (1, 1)	b) (1, 2)
c) (2, 2)	d) (3, 3)

8. If $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$, then value of a + b – c + 2d is: 1

a) 8	b) 10
c) 4	d) –8

9. The point at which the normal to the curve y = $x + \frac{1}{x}$, x > 0 is perpendicular to the line 3x – 4y – 7 = 0 is: 1

a) (2, 5/2)	b) (±2, 5/2)
c) (- 1/2, 5/2)	d) (1/2, 5/2)

10. $\sin(\tan^{-1}x)$, where |x| < 1, is equal to: 1

a) $\frac{x}{\sqrt{1-x^2}}$	b) $\frac{1}{\sqrt{1-x^2}}$
c) $\frac{1}{\sqrt{1+x^2}}$	d) $\frac{x}{\sqrt{1+x^2}}$

11. Let the relation R in the set A = {x ∈ Z : 0 ≤ x ≤ 12}, given by R = {(a, b) : |a – b| is a multiple of 4}. Then [1], the equivalence class containing 1, is: 1

a) {1, 5, 9}	b) {0, 1, 2, 5}
c) ϕ	d) A

12. If $e^x + e^y = e^{x+y}$, then $\frac{dy}{dx}$ is: 1

a) e^{y-x}	b) e^{x+y}
c) $-e^{y-x}$	d) $2e^{x-y}$

13\. Given that matrices A and B are of order 3×n and m×5 respectively, then the order of matrix C = 5A +3B is: — 1

a) 3×5	b) 5×3
c) 3×3	d) 5×5

14\. If y = 5 cos x – 3 sin x, then $\frac{d^2y}{dx^2}$ is equal to: — 1

a) - y	b) y
c) 25y	d) 9y

15\. For matrix A = $\begin{bmatrix} 2 & 5 \\ -11 & 7 \end{bmatrix}$, $(adjA)'$ is equal to: — 1

a) $\begin{bmatrix} -2 & -5 \\ 11 & -7 \end{bmatrix}$	b) $\begin{bmatrix} 7 & 5 \\ 11 & 2 \end{bmatrix}$
c) $\begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$	d) $\begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix}$

16\. The points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are parallel to y-axis are: — 1

a) $(0,\pm 4)$	b) (±4,0)
c) $(\pm 3,0)$	d) $(0, \pm 3)$

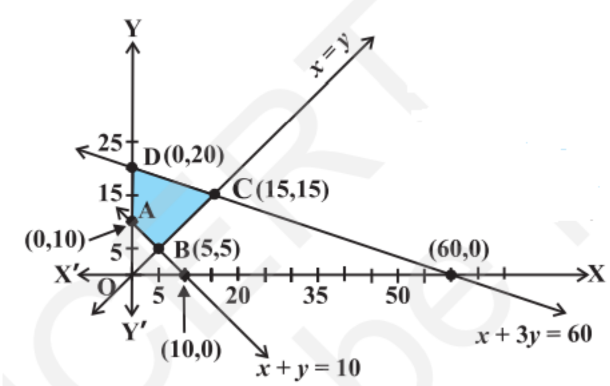
17\. Given that A = $[a_{ij}]$ is a square matrix of order 3×3 and |A| = -7, then the value of $\sum_{i=1}^{3} a_{i2}A_{i2}$, where A_{ij} denotes the cofactor of element a_{ij} is: — 1

a) 7	b) -7
c) 0	d) 49

18\. If y = log($\cos e^x$), then $\frac{dy}{dx}$ is: — 1

a) $\cos e^{x-1}$	b) $e^{-x} \cos e^x$
c) $e^x \sin e^x$	d) $- e^x \tan e^x$

19\. Based on the given shaded region as the feasible region in the graph, at which point(s) is the objective function Z = 3x + 9y maximum? — 1

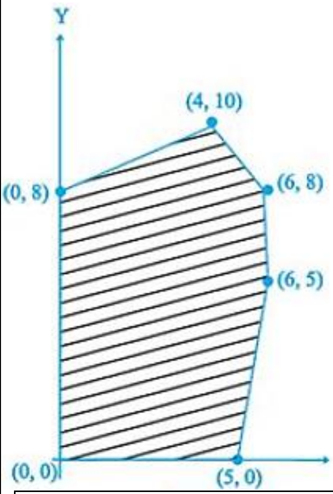


a) Point B	b) Point C
c) Point D	d) every point on the line segment CD

20.	The least value of the function $f(x) = 2cosx + x$ in the closed interval $[0,\frac{\pi}{2}]$ is: a) 2 b) $\frac{\pi}{6} + \sqrt{3}$ c) $\frac{\pi}{2}$ d) The least value does not exist.	1

SECTION – B

In this section, attempt any 16 questions out of the Questions 21 - 40.
Each Question is of 1 mark weightage.

21.	The function f: R⟶R defined as $f(x) = x^3$ is: a) One-on but not onto b) Not one-one but onto c) Neither one-one nor onto d) One-one and onto	1
22.	If x = a sec θ, y = b tan θ, then $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$ is: a) $\frac{-3\sqrt{3}b}{a^2}$ b) $\frac{-2\sqrt{3}b}{a}$ c) $\frac{-3\sqrt{3}b}{a}$ d) $\frac{-b}{3\sqrt{3}a^2}$	1
23.	 In the given graph, the feasible region for a LPP is shaded. The objective function Z = 2x – 3y, will be minimum at: a) (4, 10) b) (6, 8) c) (0, 8) d) (6, 5)	1
24.	The derivative of $\sin^{-1}(2x\sqrt{1-x^2})$ w.r.t $\sin^{-1}x$, $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$, is: a) 2 b) $\frac{\pi}{2} - 2$ c) $\frac{\pi}{2}$ d) -2	1
25.	If A = $\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}$ and B = $\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$, then: a) $A^{-1} = B$ b) $A^{-1} = 6B$ c) $B^{-1} = B$ d) $B^{-1} = \frac{1}{6}A$	1

26.	The real function f(x) = $2x^3 - 3x^2 - 36x + 7$ is: a) Strictly increasing in $(-\infty, -2)$ and strictly decreasing in $(-2, \infty)$ b) Strictly decreasing in $(-2, 3)$ c) Strictly decreasing in $(-\infty, 3)$ and strictly increasing in $(3, \infty)$ d) Strictly decreasing in $(-\infty, -2) \cup (3, \infty)$	1
27.	Simplest form of $\tan^{-1}\left(\frac{\sqrt{1+cosx}+\sqrt{1-cosx}}{\sqrt{1+cosx}-\sqrt{1-cosx}}\right), \pi < x < \frac{3\pi}{2}$ is: a) $\frac{\pi}{4} - \frac{x}{2}$ b) $\frac{3\pi}{2} - \frac{x}{2}$ c) $-\frac{x}{2}$ d) $\pi - \frac{x}{2}$	1
28.	Given that A is a non-singular matrix of order 3 such that $A^2 = 2A$, then value of $\|2A\|$ is: a) 4 b) 8 c) 64 d) 16	1
29.	The value of b for which the function $f(x) = x + cosx + b$ is strictly decreasing over **R** is: a) $b < 1$ b) No value of b exists c) $b \leq 1$ d) $b \geq 1$	1
30.	Let R be the relation in the set N given by R = {(a, b) : a = b – 2, b > 6}, then: a) (2,4) ∈ R b) (3,8) ∈ R c) (6,8) ∈ R d) (8,7) ∈ R	1
31.	The point(s), at which the function f given by $f(x) = \begin{cases} \frac{x}{\|x\|}, x < 0 \\ -1, x \geq 0 \end{cases}$ is continuous, is/are: a) $x \epsilon$ R b) $x = 0$ c) $x \epsilon$ R –{0} d) $x = -1$ and 1	1
32.	If A = $\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and kA = $\begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a and b respectively are:	1

	a) $-6, -12, -18$ b) $-6, -4, -9$ c) $-6, 4, 9$ d) $-6, 12, 18$	
33.	A linear programming problem is as follows: $Minimize\ Z = 30x + 50y$ subject to the constraints, $3x + 5y \geq 15$ $2x + 3y \leq 18$ $x \geq 0, y \geq 0$ In the feasible region, the minimum value of Z occurs at a) a unique point b) no point c) infinitely many points d) two points only	1
34.	The area of a trapezium is defined by function f and given by $f(x) = (10 + x)\sqrt{100 - x^2}$, then the area when it is maximised is: a) $75cm^2$ b) $7\sqrt{3}cm^2$ c) $75\sqrt{3}cm^2$ d) $5cm^2$	1
35.	If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to: a) A b) I + A c) I – A d) I	1
36.	If $\tan^{-1} x = y$, then: a) $-1 < y < 1$ b) $\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$ c) $\frac{-\pi}{2} < y < \frac{\pi}{2}$ d) $y \in \{\frac{-\pi}{2}, \frac{\pi}{2}\}$	1
37.	Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Based on the given information, f is best defined as: a) Surjective function b) Injective function c) Bijective function d) function	1
38.	For A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $14A^{-1}$ is given by: a) $14\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ b) $\begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$ c) $2\begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix}$ d) $2\begin{bmatrix} -3 & -1 \\ 1 & -2 \end{bmatrix}$	1
39.	The point(s) on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$ is/are: a) (-2,19) b) (2, - 9) c) $(\pm 2, 19)$ d) (-2, 19) and (2, -9)	1
40.	Given that A = $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ and $A^2 = 3I$, then:	1

a) $1+\alpha^2+\beta\gamma=0$	b) $1-\alpha^2-\beta\gamma=0$
c) $3-\alpha^2-\beta\gamma=0$	d) $3+\alpha^2+\beta\gamma=0$

SECTION – C

In this section, attempt any 8 questions.
Each question is of 1-mark weightage.
Questions 46-50 are based on a Case-Study.

41. For an objective function $Z = ax + by$, where $a, b > 0$; the corner points of the feasible region determined by a set of constraints (linear inequalities) are (0, 20), (10, 10), (30, 30) and (0, 40). The condition on *a* and *b* such that the maximum Z occurs at both the points (30, 30) and (0, 40) is: — 1

a) $b-3a=0$	b) $a=3b$
c) $a+2b=0$	d) $2a-b=0$

42. For which value of m is the line y = mx + 1 a tangent to the curve $y^2 = 4x$? — 1

a) $\frac{1}{2}$	b) 1
c) 2	d) 3

43. The maximum value of $[x(x-1)+1]^{\frac{1}{3}}$, $0 \le x \le 1$ is: — 1

a) 0	b) $\frac{1}{2}$
c) 1	d) $\sqrt[3]{\frac{1}{3}}$

44. In a linear programming problem, the constraints on the decision variables x and y are $x-3y \ge 0, y \ge 0, 0 \le x \le 3$. The feasible region — 1

a) is not in the first quadrant	b) is bounded in the first quadrant
c) is unbounded in the first quadrant	d) does not exist

45. Let A = $\begin{bmatrix} 1 & \sin\alpha & 1 \\ -\sin\alpha & 1 & \sin\alpha \\ -1 & -\sin\alpha & 1 \end{bmatrix}$, where $0 \le \alpha \le 2\pi$, then: — 1

a) |A|=0	b) |A| $\epsilon(2,\infty)$
c) |A| $\epsilon(2,4)$	d) |A| $\epsilon[2,4]$

CASE STUDY

The fuel cost per hour for running a train is proportional to the square of the speed it generates in km per hour. If the fuel costs ₹ 48 per hour at speed 16 km per hour and the fixed charges to run the train amount to ₹ 1200 per hour.

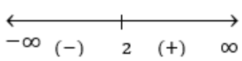
Assume the speed of the train as v km/h.

	Based on the given information, answer the following questions.	
46.	Given that the fuel cost per hour is k times the square of the speed the train generates in km/h, the value of k is: a) $\frac{16}{3}$ b) $\frac{1}{3}$ c) 3 d) $\frac{3}{16}$	1
47.	If the train has travelled a distance of 500km, then the total cost of running the train is given by function: a) $\frac{15}{16}v + \frac{600000}{v}$ b) $\frac{375}{4}v + \frac{600000}{v}$ c) $\frac{5}{16}v^2 + \frac{150000}{v}$ d) $\frac{3}{16}v + \frac{6000}{v}$	1
48.	The most economical speed to run the train is: a) 18km/h b) 5km/h c) 80km/h d) 40km/h	1
49.	The fuel cost for the train to travel 500km at the most economical speed is: a) ₹ 3750 b) ₹ 750 c) ₹ 7500 d) ₹ 75000	1
50.	The total cost of the train to travel 500km at the most economical speed is: a) ₹ 3750 b) ₹ 75000 c) ₹ 7500 d) ₹ 15000	1

Marking Scheme

Mathematics (Term-I)

Class-XII (Code-041)

Q.N.	Correct Option	Hints / Solutions
1	**d**	$sin\left(\frac{\pi}{3}-\left(\frac{-\pi}{6}\right)\right)=sin\ \left(\frac{\pi}{2}\right)=1$
2	**b**	$\lim_{x\to o}\left(\frac{1-cos\ kx}{xsin\ x}\right)=\frac{1}{2}$ $\Rightarrow \lim_{x\to o}\left(\frac{2sin^2\frac{kx}{2}}{xsin\ x}\right)=\frac{1}{2}$ $\Rightarrow \lim_{x\to o} 2\left(\frac{k}{2}\right)^2\left(\frac{sin\frac{kx}{2}}{\frac{kx}{2}}\right)^2\left(\frac{x}{sinx}\right)=\frac{1}{2}$ $\Rightarrow k^2$=1$\Rightarrow$ k $=\pm 1\ but\ k<0\ \Rightarrow$ k= -1
3	**d**	$A^2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$
4	**c**	As A is singular matrix $\Rightarrow \|A\|=0$ $\Rightarrow 2k^2-32=0\ \Rightarrow k=\pm 4$
5	**b**	$f(x)=x^2-4x+6$ $f'(x)=2x-4$ $let\ f'(x)=0\Rightarrow x=2$  as $f'(x)>0\ \ \forall\ \ x\in(2,\infty)$ $\Rightarrow f(x)$ is Strictly increasing in $(2,\infty)$
6	**d**	as $\|adj\ A\|=\|A\|^{n-1}$, where n is order of the square matrix A $=(-4)^2=16$
7	**b**	$(1,2)$
8	**a**	$2a+b=4\ a-2b=-3\ 5c-d=11\ 4c+3d=24\ \}\quad \Rightarrow a=1\ b=2\ c=3\ d=$ 4 $\therefore a+b-c+2d=8$
9	**a**	$f(x)=x+\frac{1}{x},x>0\Rightarrow f'(x)=1-\frac{1}{x^2}=\frac{x^2-1}{x^2},x>0$ As normal to the curve $y=f(x)$ at some point (x, y) is $\perp$ to given line $\Rightarrow\left(\frac{x^2}{1-x^2}\right)\times\frac{3}{4}=-1\ \ (m_1.m_2=\ -1)$ $\Rightarrow\ x^2=4\Rightarrow x=\pm 2$ But $x>0,\therefore x=2$ Therefore point=$\left(2,\ \frac{5}{2}\right)$
10	**d**	$sin\ (tan^{-1}x)$ =$sin\ \{sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\}\ =\frac{x}{\sqrt{1+x^2}}$
11	**a**	$\{1,5,9\}$
12	**c**	$e^x+e^y=e^{x+y}$ $\Rightarrow e^{-y}+e^{-x}=1$ Differentiating *w.r.t.* x:

		$\Rightarrow\ -e^{-y}\frac{dy}{dx}-e^{-x}=0 \Rightarrow \frac{dy}{dx}=-e^{y-x}$
13	**b**	3×5
14	**a**	$y=5\cos x-3\sin x\ \Rightarrow \frac{dy}{dx}=-5\sin x-3\cos x$ $\Rightarrow\ \frac{d^2y}{dx^2}=-5\cos x+3\sin x\ =\ -y$
15	**c**	$adj\ A=\begin{bmatrix}7 & -5\\ 11 & 2\end{bmatrix} \Rightarrow (adjA)'=\begin{bmatrix}7 & 11\\ -5 & 2\end{bmatrix}$
16	**c**	$\frac{x^2}{9}+\frac{y^2}{16}=1 \Rightarrow \frac{2x}{9}+\frac{2y}{16}\frac{dy}{dx}=0$ $\Rightarrow slope\ of\ normal\ at\ any\ point\ (x,y)\ to\ the\ curve=\frac{-dx}{dy}=\frac{9y}{16x}$ As tangent to the curve at the point (x, y) is parallel to *y*-axis $\Rightarrow \frac{9y}{16x}=0 \Rightarrow y=0$ and $x=\pm 3$ $\therefore\ points=(\pm 3,0)$
17	**b**	$\lvert A\rvert=-7$ $\therefore \sum_{i=1}^{3}a_{i2}A_{i2}=a_{12}A_{12}+a_{22}A_{22}+a_{32}A_{32}=\lvert A\rvert=-7$
18	**d**	$y=log\ (cos\ e^x)$ Differentiating w.r.t. x: $\frac{dy}{dx}=\frac{1}{cos\ (e^x)}.(-sin\ e^x).\ e^x$ (chain rule) $\Rightarrow \frac{dy}{dx}=-e^x\ tan\ e^x$
19	**d**	Z is maximum 180 at points *C (15,15)* and *D(0, 20).* ⇒ Z is maximum at every point on the line segment *CD*
20	**c**	$f(x)=2cosx+x\ ,\ x\in[0,\frac{\pi}{2}]$ $f'(x)=-2sinx+1$ Let$f'(x)=0 \Rightarrow x=\frac{\pi}{6}\ \in[0,\frac{\pi}{2}]$ $f(0)=2$ $f\left(\frac{\pi}{6}\right)=\frac{\pi}{6}+\sqrt{3}$ $f\left(\frac{\pi}{2}\right)=\frac{\pi}{2}$ ⇒ least value of $f(x)$ is $\frac{\pi}{2}$ at $x=\frac{\pi}{2}$
		Section-B
21	**d**	$let\ f(x_1)=f(x_2)\ such\ that\ x_1x_2\in R$ $\Rightarrow\ {x_1}^3={x_2}^3$ $\Rightarrow\ x_1=x_2$ ⇒ f is one - one $Let\ y\in R(codomain).Then\ for\ any\ x, f(x)=y\ if x^3=y$ $i.e., x=y^{\frac{1}{3}}\in R(domain)$ i.e., every element $y\ \in R(codomain)$ has a pre image $y^{\frac{1}{3}}$ in R(domain) ⇒ f is onto ∴ f is one-one and onto
22	**a**	$x=a\ sec\theta \Rightarrow \frac{dx}{d\theta}=asec\theta tan\theta$ $y=btan\ \theta\ \Rightarrow\ \frac{dy}{d\theta}=b\ sec^2\theta$ $\therefore \frac{dy}{dx}=\frac{b}{a}cosec\theta$ $\Rightarrow \frac{d^2y}{dx^2}=\frac{-b}{a}\ cosec\theta.cot\theta.\frac{d\theta}{dx}=\frac{-b}{a^2}cot^3\theta$ $\therefore\ \frac{d^2y}{dx^2}\Big]_{\theta=\frac{\pi}{6}}=\frac{-3\sqrt{3}b}{a^2}$
23	**c**	Z is minimum -24 at (0, 8)
24	**a**	$let\ u=sin^{-1}(2x\sqrt{1-x^2}\ )$

		and $v = sin^{-1}x,\ -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ $\Rightarrow sin\, v = x$ ……(1) Using (1), we get : $=sin^{-1}(2sin\, v\, cos\, v) = sin^{-1}(sin2\, v)$ $\Rightarrow u = 2v, -\frac{\pi}{2} < 2v < \frac{\pi}{2}$ Differentiating u with respect to v, we get: $\frac{du}{dv} = 2$
25	**d**	$AB = 6I \Rightarrow B^{-1} = \frac{1}{6}A$
26	**b**	$f'(x) = 6(x^2 - x - 6) = 6(x-3)(x+2)$ As $f'(x) < 0 \ \forall\ x \in (-2,3)$ $\Rightarrow$ $f(x)$ is strictly decreasing in $(-2,3)$
27	**a**	$tan^{-1}\left(\frac{\sqrt{1+cos\,x}+\sqrt{1-cosx}}{\sqrt{1+cos\,x}-\sqrt{1-cos\,x}}\right)$ $= tan^{-1}\left(\frac{-\sqrt{2}cos\frac{x}{2}+\sqrt{2}sin\frac{x}{2}}{-\sqrt{2}cos\frac{x}{2}-\sqrt{2}sin\frac{x}{2}}\right)$, $\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$ $= tan^{-1}\left(\frac{cos\frac{x}{2}-sin\frac{x}{2}}{cos\frac{x}{2}+sin\frac{x}{2}}\right) = tan^{-1}\left(\frac{1-tan\frac{x}{2}}{1+tan\frac{x}{2}}\right) = tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$ $= \frac{\pi}{4} - \frac{x}{2}, -\frac{\pi}{4} > \frac{\pi}{4} - \frac{x}{2} > -\frac{\pi}{2}$
28	**c**	$A^2 = 2A$ $\Rightarrow \left\|A^2\right\| = \|2A\|$ $\Rightarrow \|A\|^2 = 2^3\|A\|$ as $\|kA\| = k^n\|A\|\ for\ a\ square\ matrix\ of\ order\ n$ $\Rightarrow$ either $\|A\| = 0\ or\ \|A\| = 8$ But A is non-singular matrix $\therefore \|A\| = 8^2 = 64$
29	**b**	$f'(x) = 1 - sin\,x \Rightarrow f'(x) \geq 0\ \ \forall x \in R$ $\Rightarrow no\ value\ of\ b\ exists$
30	**c**	$a = b - 2\ and\ b > 6$ $\Rightarrow (6,8) \in R$
31	**a**	$f(x) = \{\frac{x}{-x} = -1\ ,\quad x < 0 \quad -1\ ,\quad x \geq 0$ $\Rightarrow f(x) = -1\ \forall\ x \in R$ $\Rightarrow f(x) is\ continous\ \forall\ x \in R\ as\ it\ is\ a\ constant\ function$
32	**b**	$kA = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$ $\Rightarrow k = -6, a = -4\ and\ b = -9$
33	**c**	Corner points of feasible region $Z = 30x + 50y$ (5,0) 150 (9,0) 270 (0,3) 150 (0,6) 300 Minimum value of Z occurs at infinitely many points
34	**c**	$f'(x) = \frac{-2x^2-10x+100}{\sqrt{100-x^2}}$ $f'(x) = 0 \Rightarrow x = -10\ or\ 5$, But $x > 0 \Rightarrow x = 5$ $f''(x) = \frac{2x^3-300x-1000}{(100-x)^{\frac{3}{2}}} \Rightarrow f''(5) = \frac{-30}{\sqrt{75}} < 0$ $\Rightarrow$ Maximum area of trapezium is $75\sqrt{3}\ cm^2$ when x = 5
35	**d**	$(I-A)^3 - 7A = I + A + 3A + 3A - 7A = I$
36	**c**	$\frac{-\pi}{2} < y < \frac{\pi}{2}$

37	**b**	Since, distinct elements of A have distinct f-images in B. Hence, f is injective and every element of B does not have its pre-image in A, hence f is not surjective. $\therefore f \text{ is injective and is not surjective}.$
38	**b**	$\lvert A\rvert = 7\ ,\ adjA = \begin{bmatrix}2 & -1\\1 & 3\end{bmatrix}$ $\therefore 14A^{-1} = 14 \times \frac{1}{7}\begin{bmatrix}2 & -1\\1 & 3\end{bmatrix} = \begin{bmatrix}4 & -2\\2 & 6\end{bmatrix}$
39	**b**	$y = x^3 - 11x + 5 \ \Rightarrow \frac{dy}{dx} = 3x^2 - 11$ Slope of line $y = x - 11$ is $1 \ \Rightarrow 3x^2 - 11 = 1 \Rightarrow x = \pm 2$ ∴ point is (2, -9) as (-2, 19) does not satisfy the equation of the given line
40	**c**	$A^2 = 3I$ $\Rightarrow \begin{bmatrix}\alpha^2 + \beta\gamma & 0\\0 & \beta\gamma + \alpha^2\end{bmatrix} = \begin{bmatrix}3 & 0\\0 & 3\end{bmatrix} \quad \Rightarrow 3 - \alpha^2 - \beta\gamma = 0$
		Section C
41	**a**	As Z is maximum at (30, 30) and (0, 40) $\Rightarrow 30a + 30b = 40b \Rightarrow b - 3a = 0$
42	**b**	$y = mx + 1 \ldots\ldots(1)$ and $y^2 = 4x \ldots\ldots(2)$ Substituting (1) in (2) : $(mx + 1)^2 = 4x$ $\Rightarrow m^2x^2 + (2m - 4)x + 1 = 0 \ldots\ldots\ldots(3)$ As line is tangent to the curve ⇒ line touches the curve at only one point $\Rightarrow (2m - 4)^2 - 4m^2 = 0 \Rightarrow m = 1$
43	**c**	Let $f(x) = [x(x - 1) + 1]^{\frac{1}{3}}, \quad 0 \le x \le 1$ $f'(x) = \frac{2x-1}{3(x^2-x+1)^{\frac{2}{3}}}$ let $f'(x) = 0 \quad \Rightarrow \ x = \frac{1}{2} \in [0, 1]$ $f(o) = 1, f\left(\frac{1}{2}\right) = \left(\frac{3}{4}\right)^{\frac{1}{3}}$ and $f(1) = 1$ ∴ Maximum value of $f(x)$ is 1
44	**b**	Feasible region is bounded in the first quadrant
45	**d**	$\lvert A\rvert = 2 + 2sin^2\alpha$ As $-1 \le sin\alpha \le 1, \forall\ 0 \le \alpha \le 2\pi$ $\Rightarrow 2 \le 2 + 2sin^2\alpha \le 4 \Rightarrow \lvert A\rvert \in [2,4]$
46	**d**	Fuel cost per hour = $k(speed)^2$ $\Rightarrow 48 = k.16^2 \Rightarrow k = \frac{3}{16}$
47	**b**	Total cost of running train (let C) =$\frac{3}{16}v^2t + 1200t$ Distance covered = 500km ⇒ $time = \frac{500}{v}hrs$ Total cost of running train 500 km= $\frac{3}{16}v^2\left(\frac{500}{v}\right) + 1200\left(\frac{500}{v}\right)$ $\Rightarrow \ C = \frac{375}{4}v + \frac{600000}{v}$
48	**c**	$\frac{dC}{dv} = \frac{375}{4} - \frac{600000}{v^2}$ Let $\frac{dC}{dv} = 0 \quad \Rightarrow v = 80\ km/h$
49	**c**	Fuel cost for running 500 km $\frac{375}{4}v = \frac{375}{4} \times 80 = Rs.7500/-$
50	**d**	Total cost for running 500 km = $\frac{375}{4}v + \frac{600000}{v}$ $= \frac{375\times80}{4} + \frac{600000}{80} = Rs.15000/-$